

RUBRICS: HSSC ANNUAL EXAMINATION 2023
SUBJECT: MATHEMATICS HSSC-II (HA)

Q.# /Part #	Criteria	Level 1 (Marks)	Level 2(Marks)	Level 3 (Marks)	Level 4 (Marks)	Level 5 (Marks)	Level 6 (Marks)
2(i)	Finding values of $f \circ g(x)$ and x for which $g \circ f(x) = 7$	(a) Correctly finding the value of $f \circ g(x)$. (2)	Partially correct (1)	Wrong Answer (0)			
		(b) Correctly stating $g(1 + 2x)$ (1)	Partially correct (0.5)	Wrong Answer (0)			
		(b) Finding the correct value of x . (1)	Partially correct (0.5)	Wrong Answer (0)			
2(ii)	Explaining the existence of limit and discussing the continuity of $f(x)$ at $x = 1$.	(a) Correctly finding Left Limit AND Right Limit at $x = 1$. (2)	Correctly finding Left Limit OR Right Limit at $x = 1$. (1)	Partially correct response (0.5)	Wrong response (0)		
		(b) Correctly finding value, Left Limit and Right Limit of $f(x)$ at $x = 1$. (1.5)	Any two correct aspects. (1)	Any one correct aspect. (0.5)	Wrong Answer (0)		
		(b) Correctly stating $f(x)$ continuous at $x = 1$. (0.5)	Wrong Answer (0)				
2(iii)	Differentiating $y = \cos^{-1} \left(\frac{x^2-1}{x^2+1} \right)$ w. r. t. x	Correctly applying the derivative of inverse cosine function AND correctly applying the quotient law of derivatives. (2)	Correctly applying the derivative of inverse cosine function OR correctly applying the quotient law of derivatives. (1)	Partially correct (0.5)	Wrong Answer (0)		
		Correctly writing the answer in the simplified form. (2)	Partially correct (1)	Wrong Answer (0)			
2(iv)	Finding the value of $\frac{dy}{dx}$ at $x = 1$.	Correctly differentiating the given equation and making $\frac{dy}{dx}$ subject. (2)	Correctly differentiating the given equation not making $\frac{dy}{dx}$ subject. (2)	Partially correct (1)	Wrong Answer (0)		
		Correctly finding the value of $\frac{dy}{dx}$ at $x = 1$. (2)	Partially correct (0.5)	Wrong Answer (0)			

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2(v)	Showing the expansion of e^{x+h} by using Taylor's series.	Correctly finding the four derivatives of $f(x)$ (3)	Correctly finding any three derivatives of $f(x)$ (2.5)	Correctly finding any two derivatives of $f(x)$. (2)	Correctly finding any one derivatives of $f(x)$. (1)	Wrong Answer (0)	
		Correctly stating AND correctly showing the Taylor's Series expansion (1).	Correctly stating OR correctly showing the Taylor's Series expansion (0.5).	Wrong Answer (0)			
2(vi)	Applying second derivative test to find the dimensions of a rectangular garden having maximum area.	Correctly writing a function for extreme values AND Correctly finding the 1 st and 2 nd derivative of the function. (2)	Correctly writing a function for extreme values OR Correctly finding the 1 st and 2 nd derivative of the function. (1)	Partially correct (0.5)	Wrong Answer (0)		
		Setting 1 st derivative zero and finding the correct value of x (1)	Setting 1 st derivative zero and finding the incorrect value of x (0.5)	Wrong Answer (0)			
		Correctly applying the 2 nd derivative test and finding the correct dimensions. (1)	Correctly applying the 2 nd derivative test and finding the incorrect dimensions. (0.5)	Wrong Answer (0)			
2(vii)	Finding the approximate increase in the volume of a cube using differentials	Correctly stating volume of the cube AND Correctly measuring change in length of the cube (2)	Correctly stating volume of the cube OR Correctly measuring change in length of the cube (1)	Partially correct (0.5)	Wrong Answer (0)		
		Correctly finding differential of the volume function AND Correctly finding the approximate increase in volume (2)	Correctly finding differential of the volume function OR Correctly finding the approximate increase in volume (1)	Partially correct (0.5)	Wrong Answer (0)		
2 (viii)	Evaluating the indefinite integral	Correctly formulating the integral (2)	Partially correct (1)	Wrong Answer (0)			
		Integrating correctly (2)	Partially correct (1)	Wrong Answer (0)			

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2(ix)	Evaluating the definite integral	Correctly integrating $\cos\left(\frac{x}{2}\right)$ (2)	Partially correct (1)	Wrong Answer (0)			
		Correctly evaluating boundary limits. (2)	Partially correct (1)	Wrong Answer (0)			
2(x)	Solving the differential equation at (2,1)	Separating the variables correctly (1)	Partially correct (1)	Wrong Answer (0)			
		Correctly integrating LHS and RHS (2)	Integrating any of the side correctly (1)	Wrong Answer (0)			
		Finding the correct solution (1)	Partially correct (0.5)	Wrong Answer (0)			
2(xi)	Finding the equation of straight line with given conditions.	Correctly stating equation of a straight line in two intercepts form. (2)	Partially correct (1)	Wrong Answer (0)			
		Correctly finding the intercept at (4, 4) AND Correctly finding equation of the straight line (2)	Correctly finding the intercept at (4, 4) OR Correctly finding equation of the straight line. (1)	Partially correct (0.5)	Wrong Answer (0)		
2(xii)	Applying the condition of concurrency of three straight lines and finding the value of k .	Correctly applying the condition of concurrency of three straight lines. (2)	Partially correct (1)	Wrong Answer (0)			
		Correctly expanding the determinant AND correctly finding the value of k . (2)	Correctly expanding the determinant OR correctly finding the value of k . (1)	Partially correct (0.5)	Wrong Answer (0)		
2(xiii)	Finding the equation of a circle passing through the given three points.	Correctly stating the circle equation. (1)	Wrong Answer (0)				
		Correctly finding center AND radius of the circle. (2)	Correctly finding center OR radius of the circle (1)	Partially correct (0.5)	Wrong Answer (0)		
		Correctly finding the circle equation. (1)	Partially correct (0.5)	Wrong Answer (0)			

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2(xiv)	Finding height of the suspension bridge at a point 150m from the center of the base.	Correctly writing equation of the parabolic suspension bridge. $(x^2 = 4ay)$ (1)	Partially correct (0.5)	Wrong Answer (0)			
		Correctly finding distance 'a' between focus and vertex of the arch. (2)	Partially correct (1)	Wrong Answer (0)			
		Correctly finding the required height of the arch. (1)	Partially correct (0.5)	Wrong Answer (0)			
2(xv)	Finding a unit vector \perp to the plane containing \underline{a} and \underline{b} . Verifying that vectors \underline{a} and \underline{b} are perpendicular to $\underline{a} \times \underline{b}$.	Correctly finding $\underline{a} \times \underline{b}$ AND Correctly finding a unit vector along $\underline{a} \times \underline{b}$. (2)	Correctly finding $\underline{a} \times \underline{b}$ OR Correctly finding a unit vector along $\underline{a} \times \underline{b}$. (1)	Partially correct (0.5)	Wrong Answer (0)		
		Correctly verifying that $\underline{a} \perp (\underline{a} \times \underline{b})$ AND $\underline{b} \perp (\underline{a} \times \underline{b})$ (2)	Correctly verifying that $\underline{a} \perp (\underline{b} \times \underline{a})$ OR $\underline{b} \perp (\underline{b} \times \underline{a})$ (1)	Partially correct (0.5)	Wrong Answer (0)		
2(xvi)	In any triangle ABC, Proving that $c^2 = a^2 + b^2 - 2ab\cos C$	Correctly stating $\underline{a} + \underline{b} + \underline{c} = 0$ AND Correctly stating $\underline{c} \cdot \underline{c} = (-\underline{a} - \underline{b}) \cdot (-\underline{a} - \underline{b})$ (2)	Correctly stating $\underline{a} + \underline{b} + \underline{c} = 0$ OR Correctly stating $\underline{c} \cdot \underline{c} = (-\underline{a} - \underline{b}) \cdot (-\underline{a} - \underline{b})$ (1)	Partially correct (0.5)	Wrong Answer (0)		
		Correctly verifying $c^2 = a^2 + b^2 - 2ab\cos C$ (2)	Partially correct (1)	Wrong Answer (0)			
3	Evaluating Left and Right limits of $f(x)$. Finding values of a and b for $f(x)$ being continuous at $x = 1$ Sketching graph of $f(x)$.	(a) Correctly finding Left Limit AND Right Limit. (2)	Correctly finding Left Limit OR Right Limit. (1)	Partially correct response (0.5)	Wrong response (0)		
		(b) Correctly applying the conditions of continuity of the function AND Correctly finding a pair of linear equations in a and b . (2)	Correctly applying the conditions of continuity of the function OR Correctly finding a pair of linear equations. (1)	Partially correct (0.5)	Wrong Answer (0)		
		(c) Correctly solving pair of linear equations for the values of a AND b . (2)	Correctly solving pair of linear equations for the values of a OR b . (1)	Partially correct (0.5)	Wrong Answer (0)		

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		(d) Correctly tabulating AND sketching the graph of $f(x)$. (2)	Correctly tabulating OR sketching the graph of $f(x)$. (1)	Partially correct (0.5)	Wrong Answer (0)		
4	Expressing total surface area A as function of its side x Finding $\frac{dA}{dx}$ and $\frac{d^2A}{dx^2}$ Finding dimensions of the rectangular block having minimum surface area.	(a) Correctly expressing A in terms of x, y AND Correctly expressing A in terms of x . (2)	Correctly expressing A in terms of x, y AND Correctly expressing A in terms of x . (1)	Partially correct (0.5)	Wrong Answer (0)		
		(b) Correctly finding the values of $\frac{dA}{dx}$ AND $\frac{d^2A}{dx^2}$ (2)	Correctly finding the values of $\frac{dA}{dx}$ OR $\frac{d^2A}{dx^2}$ (1)	Partially correct (0.5)	Wrong Answer (0)		
		(c) Setting $\frac{dA}{dx} = 0$ AND correctly finding the critical value of x . (2)	Setting $\frac{dA}{dx} = 0$ OR correctly finding the critical value of x . (1)	Partially correct (0.5)	Wrong Answer (0)		
		(c) Correctly applying the 2 nd derivative test AND finding the correct dimensions of the block. (2)	Correctly applying the 2 nd derivative test OR finding the correct dimensions of the block (1)	Partially correct (0.5)	Wrong Answer (0)		
5	Evaluating the definite integral. Sketching the graph of $f(x)$. Shading and finding the bounded area.	(a) Correctly integrating $f(x)$ AND evaluating the boundary limits. (2)	Correctly integrating $f(x)$ OR evaluating the boundary limits. (1)	Partially correct (0.5)	Wrong Answer (0)		
		(b) Correctly tabulating AND Correctly sketching the graph of $f(x)$. (2)	Correctly tabulating OR Correctly sketching the graph of $f(x)$. (1)	Partially correct (0.5)	Wrong Answer (0)		
		(b) Correctly shading the bounded region of $f(x)$. (1)	Partially correct (0.5)	Wrong Answer (0)			

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		(c) Correctly stating the bounded area in terms of two definite integrals AND Correctly evaluating the bounded area in terms of two definite integrals. (3)	Correctly stating the bounded area in terms of two definite integrals OR Evaluating partially correct bounded area in terms of two definite integrals (2)	Correctly stating the bounded area in terms of two definite integrals OR Correctly evaluating the bounded area in terms of two definite integrals (1.5)	Any one correct aspect. (1)	Partially correct (0.5)	Wrong Answer (0)
6	Finding mid points, slopes and equations of right bisectors of the sides and area of ΔABC Showing right bisectors concurrent.	(a) Correctly finding mid points and slopes of the three sides of ΔABC . (3)	Any two correct aspects. (2)	Any one correct aspect. (1)	Partially correct (0.5)	Wrong Answer (0)	
		(b) Correctly finding equations of the three right bisectors of ΔABC . (3)	Any two correct aspects. (2)	Any one correct aspect. (1)	Partially correct (0.5)	Wrong Answer (0)	
		(c) Correctly verifying the condition of concurrency of three straight lines. (1)	Partially correct (0.5)	Wrong Answer (0)			
		Correctly finding area of ΔABC . (1)	Partially correct (0.5)	Wrong Answer (0)			
7	Minimizing the cost function by Linear Programming	Correctly stating the cost function AND correctly finding the constraints. (2)	Correctly stating the profit function OR correctly finding the constraints. (1)	Partially correct (0.5)	Wrong Answer (0)		
		Correctly finding the intercepts. (1)	Partially correct (0.5)	Wrong Answer (0)			
		Correctly sketching two lines with correct point of intersection and correctly shading the feasible region (4)	Correctly sketching two lines with correct point of intersection. (3)	Correctly sketching two straight lines (2)	Correctly sketching one straight line (1)	No correct aspect (0)	
		Correctly finding the number of units to get minimum cost. (1)	Partially correct (0.5)	Wrong Answer (0)			

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8	Finding coordinates of center, vertices, and foci; value of eccentricity and equations of directrices of the given ellipse.	Correctly writing the equation of horizontal ellipse in standard form (2)	Partially correct (1)	Wrong Answer (0)			
		Correctly finding the values of a, b AND c. (1)	Correctly finding the values of a, b OR c. (0.5)	Wrong Answer (0)			
		Correctly finding coordinates of center, vertices, foci; value of eccentricity and equations of directrices. (5)	Any four correct aspects. (4)	Any three correct aspects. (3)	Any two correct aspects. (2)	Any one correct aspect. (1)	No correct aspect (0)

Note: All the markers must know the solutions of all the question items of the question paper before starting marking.