

Q. No. 2 Part (i) (Page 1)

$$f^{-1}(x) = ?$$

$$f(x) = \frac{3x+2}{2x-1}$$

$$\text{Let } f(x) = y$$

$$x = f^{-1}(y)$$

$$y = \frac{3x+2}{2x-1}$$

$$(3x+2) = (2x-1)y$$

$$3x+2 = 2xy-y$$

$$3x-2xy = -2-y$$

$$2xy-3x = y+2$$

$$x(2y-3) = y+2$$

$$x = \frac{y+2}{2y-3}$$

$$f^{-1}(y) = \frac{y+2}{2y-3}$$

$$f^{-1}(x) = \frac{x+2}{2x-3}$$

$$f^{-1}(f(x)) = x$$

$$f^{-1}(x) = \frac{x+2}{2x-3}$$

$$f(x) = \frac{3x+2}{2x-1}$$

$$f^{-1}(f(x)) = \frac{\frac{3x+2}{2x-1} + 2}{2\left(\frac{3x+2}{2x-1}\right) - 3}$$

$$= \frac{3x+2+2(2x-1)}{2(3x+2)-3(2x-1)}$$

$$= \frac{3x+2+4x-2}{6x+4-6x+3}$$

$$= \frac{7x}{6x+4-6x+3}$$

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$$f^{-1}(f(x)) = \frac{3x+2+2(2x-1)}{2(3x+2)-3(2x-1)}$$

$$= \frac{3x+2+4x-2}{6x+4-6x+3}$$

$$= \frac{7x}{6x+4-6x+3}$$

$$= \frac{7x}{6x+4-6x+3}$$

~~A~~

$$f^{-1}(f(x)) = x$$

Hence proved.

Q. No. 2 Part (ii) (Page 1)

Continuity of function :

$$f(x) = \begin{cases} 3x-1 & x < 1 \\ 4 & x = 1 \\ 2(x) & x > 1 \end{cases}$$

i- $f(1) = 4$

ii- limit

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$$

$$\lim_{x \rightarrow 1^-} (3x-1) = \lim_{x \rightarrow 1^+} (2x)$$

$$3(1) - 1 = 2(1)$$

$$3 - 1 = 2$$

$$2 = 2$$

As $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$

Limit exists.

iii. $f(1) \neq \lim_{x \rightarrow 1} f(x)$

$$4 \neq 2$$

Hence the function is discontinuous at $x=1$ since $f(1) \neq \lim_{x \rightarrow 1} f(x)$.

$$y = \cot(q \cot^{-1}(x))$$

$$\cot^{-1}(y) = q \cot^{-1}(x)$$

Taking derivative on BHS.

$$\frac{1}{1+y^2} \times \frac{dy}{dx} = q \left(\frac{1}{1+x^2} \right)$$

$$\frac{1}{(1+y^2)} \times y' = q \left(\frac{1}{1+x^2} \right)$$

Cross multiplying.

$$(1)(1+x^2)y' = q(1+y^2)$$

$$(1+x^2)y' - q(1+y^2) = 0$$

Hence proved.

$$\sin(61^\circ) = ?$$

$$\text{Let } y = \sin(x)$$

$$\text{where } x = 60^\circ \quad \text{and } x + \delta x = 61^\circ \quad \delta x = 1^\circ$$

$$y = \sin x$$

Taking differential on BHS

$$dy = \cos x \, dx$$

$$dy = \cos(60)(1^\circ)$$

$$1^\circ = 0.01745 \text{ rad.}$$

$$dy = \cos(60) \times 0.01745$$

$$dy = 8.725 \times 10^{-3}$$

$$y = \sin(x)$$

$$y + \delta y = \sin(x + \delta x)$$

$$\sin(60 + 1) = \sin(60) + 8.725 \times 10^{-3}$$

$$\sin(61^\circ) = 0.87475$$

Q. No. 2 Part (v) (Page 1)

Area bounded by curve = ?

$$y = x^3 - 9x$$

$$\text{Area} = \int_a^b y dx$$

for finding range

$$x^3 - 9x = 0$$

$$x(x^2 - 9) = 0$$

$$x = 0$$

$$x^2 = 9$$

$$x = 0$$

$$x = \pm 3.$$

$$x^3 - 9x \geq 0 \quad [-3, 0] \quad \text{graph is above } x\text{-axis}$$

$$x^3 - 9x \leq 0 \quad [0, 3] \quad \text{graph is below } x\text{-axis.}$$

$$\text{Area} = \int_{-3}^0 (x^3 - 9x) dx - \int_0^3 (x^3 - 9x) dx.$$

$$= \left| \frac{x^4}{4} - \frac{9x^2}{2} \right|_{-3}^0 - \left| \frac{x^4}{4} - \frac{9x^2}{2} \right|_0^3$$

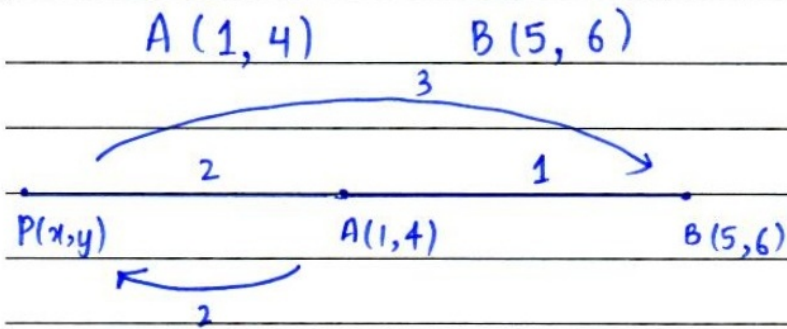
$$= \left[0 - 0 - \left[\frac{(-3)^4}{4} - \frac{9(-3)^2}{2} \right] \right] - \left[\frac{3^4}{4} - \frac{9(3)^2}{2} - 0 - 0 \right]$$

$$= - \left[\frac{-81}{4} \right] - \left[\frac{-81}{4} \right]$$

$$= \frac{81}{4} + \frac{81}{4} = \frac{81}{2} \text{ square units.}$$

Q. No. 2 Part (vi) (Page 1)

Ratio formula:



$$k_1 : k_2 = 2 : 3$$

$$A(x_1, y_1) = A(1, 4)$$

$$k_1 = 2$$

$$k_2 = 3$$

$$B(x_2, y_2) = B(5, 6)$$

$$P(x, y) = \left(\frac{k_1 x_2 + k_2 x_1}{k_1 + k_2}, \frac{k_1 y_2 + k_2 y_1}{k_1 + k_2} \right)$$

$$P(x, y) = \left(\frac{(2)(5) + 3(1)}{2+3}, \frac{(2)(6) + (3)(4)}{2+3} \right)$$

$$P(x, y) = \left(\frac{10+3}{5}, \frac{12+12}{5} \right)$$

$$P(x, y) = \left(\frac{13}{5}, \frac{24}{5} \right)$$

Q. No. 2 Part (vii) (Page 1)

Equation of line = ?

The normal form of equation of st. line is:

$$x \cos d + y \sin d = p$$

where $p =$ perpendicular distance from origin $= 8$
 $d =$ angle from +ve x -axis. $= 30^\circ$

$$x \cos(30^\circ) + y \sin(30^\circ) = 8$$

$$x \left(\frac{\sqrt{3}}{2} \right) + y \left(\frac{1}{2} \right) = 8$$

$$\frac{\sqrt{3}x}{2} + \frac{y}{2} = 8$$

$$\frac{\sqrt{3}x}{2} + \frac{y}{2} = \frac{16}{2}$$

$$\sqrt{3}x + y = 16$$

$$y = -\sqrt{3}x + 16$$

Comparing with:

$$y = mx + c$$

$$m = -\sqrt{3} \quad c = 16$$

Q. No. 2 Part (vii) (Page 2)

A series of horizontal lines for writing.

Graph:

$$3x + 2y \geq 6$$

$$x + y \leq 4$$

$$x \geq 0, y \geq 0.$$

Associated equations are:

$$3x + 2y = 6$$

$$x + y = 4$$

For x -intercept ; $y = 0$

$$3x + 0 = 6$$

$$x + 0 = 4$$

$$x = 2$$

$$x = 4$$

$$(2, 0)$$

$$(4, 0)$$

For y -intercept ; $x = 0$.

$$0 + 2y = 6$$

$$0 + y = 4$$

$$y = 3$$

$$y = 4$$

$$(0, 3)$$

$$(0, 4)$$

Test point

$$0 + 0 > 6$$

$$0 + 0 \leq 4$$

(False)

(True)

Corner points are

$$(0, 4), (0, 3), (4, 0), (2, 0)$$

Q. No. 2 Part (ix) (Page 1)

$$F(-3, 4)$$

$$\text{directrix} : 3x + 2y - 3 = 0$$

An parabola

$$\frac{|PF|}{|PM|} = 1 \quad \text{where } P(x, y) \text{ be a point on it.}$$

$$\frac{|PF|}{|PM|} = 1$$
$$\sqrt{(x+3)^2 + (y-4)^2} = \frac{|3x+2y-3|}{\sqrt{3^2+2^2}}$$

Taking square on BHS.

$$x^2 + 6x + 9 + y^2 - 8y + 16 = \frac{(9x^2 + 4y^2 + 9 + 12xy - 18x - 12y)}{(\sqrt{13})^2}$$

$$13x^2 + 78x + 117 + 13y^2 - 104y + 208 = 9x^2 + 4y^2 + 9 + 12xy - 18x - 12y = 0$$

$$13x^2 - 9x^2 + 13y^2 - 4y^2 + 78x + 18x - 104y + 12y - 12xy + 117 - 9 = 0$$
$$4x^2 + 9y^2 + 96x - 92y - 12xy + 108 = 0$$

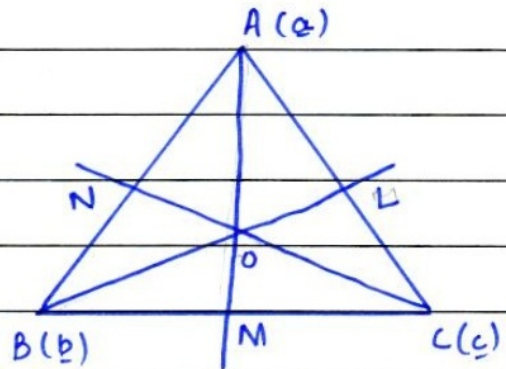
Q. No. 2 Part (x) (Page 1) Altitudes are concurrent.

Consider a triangle ABC

$$\vec{OA} = \underline{a}$$

$$\vec{OB} = \underline{b}$$

$$\vec{OC} = \underline{c}$$



$$\vec{AB} = \underline{b-a}$$

$$\vec{BC} = \underline{c-b}$$

$$\vec{AC} = \underline{a-c}$$

As we know

$$\vec{AM} \perp \vec{BC}$$

$$\vec{AO} \perp \vec{BC}$$

$$(\vec{AO}) \cdot (\vec{BC}) = 0$$

$$a \cdot (c-b) = 0$$

$$\underline{ac - ab} = 0 \quad \text{--- (1)}$$

$$\vec{CN} \perp \vec{AB}$$

$$\vec{CO} \perp \vec{AB}$$

$$\vec{CO} \cdot (\vec{AB}) = 0$$

$$c \cdot (b-a) = 0$$

$$\underline{cb - ca} = 0 \quad \text{--- (2)}$$

Adding (1) and (2)

$$\cancel{ac - bc} + \cancel{bc - ac} = 0 \quad \underline{ac - ab} + \underline{bc - ac} = 0$$

$$a(\underline{c - b}) = 0$$

$$\vec{OB} \cdot (\vec{AC}) = 0$$

$$\vec{OB} \perp \vec{AC}$$

This proves that altitudes pass through O hence are concurrent.

Q. No. 2 Part (xi) (Page 1)

Point of intersection:

$$\frac{x^2}{18} + \frac{y^2}{8} = 1 \quad \text{--- (1)}$$

$$\frac{x^2}{3} - \frac{y^2}{3} = 1 \quad \text{--- (2)}$$

Multiplying (2) by $\frac{1}{6}$

$$\left(\frac{x^2}{3} - \frac{y^2}{3} = 1 \right) \times \frac{1}{6}$$

$$\frac{x^2}{18} - \frac{y^2}{18} = \frac{1}{6} \quad \text{--- (3)}$$

Subtracting (3) and (1)

$$\begin{array}{r} \frac{x^2}{18} + \frac{y^2}{8} = 1 \\ \textcircled{+} \frac{x^2}{18} - \frac{y^2}{18} = \frac{1}{6} \\ \hline \end{array}$$

$$\frac{y^2}{8} + \frac{y^2}{18} = 1 - \frac{1}{6}$$

$$\frac{13}{72} y^2 = \frac{5}{6}$$

$$y^2 = \frac{5}{6} \times \frac{72}{13} = \frac{60}{13} \quad y = \pm \frac{2\sqrt{15}}{\sqrt{13}}$$

$$\frac{x^2}{3} - \frac{60/13}{3} = 1$$

$$\frac{x^2}{3} = 1 + \frac{60}{13(3)}$$

$$\frac{x^2}{3} = \frac{33}{13} = \frac{99}{13} \quad x = \pm \frac{3\sqrt{11}}{\sqrt{13}}$$

$$P(x, y) = P\left(\pm \frac{3\sqrt{11}}{\sqrt{13}}, \pm \frac{2\sqrt{15}}{\sqrt{13}}\right)$$

Q. No. 2 Part (xi) (Page 2) _____

Lined area for writing the answer to Q. No. 2 Part (xi).

Moment of force = ?

$$\vec{F}_1 = \hat{i} - 2\hat{j}$$

$$\vec{F}_2 = 3\hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{F}_3 = 5\hat{j} + 2\hat{k}$$

$$\begin{aligned} \vec{F}_1 + \vec{F}_2 + \vec{F}_3 &= \hat{i} - 2\hat{j} + 3\hat{i} + 2\hat{j} - \hat{k} + 5\hat{j} + 2\hat{k} \\ &= (3+1+0)\hat{i} + 5\hat{j} + (-1+2)\hat{k} \end{aligned}$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 4\hat{i} + 5\hat{j} + \hat{k}$$

$$O(1, 1, 1)$$

$$P(2, 0, 1)$$

$$\vec{OP} = [2-1]\hat{i} + [0-1]\hat{j} + [1-1]\hat{k}$$

$$\vec{OP} = \hat{i} - \hat{j} + 0$$

$$\vec{r} = \vec{OP} = \hat{i} - \hat{j}$$

$$\text{Moment of force} = \vec{r} \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 4 & 5 & 1 \end{vmatrix}$$

$$= \hat{i}(-1+0) - \hat{j}(1+0) + \hat{k}(5+4)$$

$$= -1\hat{i} - 1\hat{j} + 9\hat{k}$$

$$\mu = -\hat{i} - \hat{j} + 9\hat{k}$$

Let $l=w=x$ and $h=y$

$$V = 32 \text{ dm}^3$$

$$x \cdot x \cdot y = 32$$

$$x^2 \cdot y = 32$$

$$y = \frac{32}{x^2}$$

$$\text{Area} = x^2 + 4xy$$

$$= x^2 + 4(x) \left(\frac{32}{x^2} \right)$$

$$= x^2 + \frac{128}{x}$$

$$f(x) = x^2 + 128x^{-1}$$

$$f'(x) = 2x - 128x^{-2}$$

$$f''(x) = 2 + 2 \times 128x^{-3}$$

$$= 2 + 2 \times 128(x)^{-3}$$

$$= 2 + 256x^{-3}$$

For critical point.

$$f'(x) = 0$$

$$2x - \frac{128}{x^2} = 0$$

$$2x^3 - 128 = 0$$

$$2(x^3 - 64) = 0$$

$$(x^3 - 64) = 0$$

$$(x^3 - 8^3) = 0$$

$$(x^3 - 4^3) = 0$$

$$(x-8)(x^2+2x+4) = 0$$

$$(x-4)(x^2+4x+16) = 0$$

$$(x-4) = 0$$

$$x^2 + 4x + 16 = 0$$

$$x = 0 + 4$$

$$\boxed{x = 4}$$

Placing in $f''(x)$

$$f''(4) = 2 + \frac{256}{4^3}$$

$$= 2 + \frac{256}{64}$$

$$= 2 + 4 = 6 > 0$$

$$= 6 > 0$$

$$f''(4) = 2 + 4 = 6 > 0$$

As $f''(4)$ is +ve so

the least value will be acquired.

$$x^2 + 4x + 16 = 0$$

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(16)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16 - 64}}{2}$$

$$= \frac{-4 \pm \sqrt{-48}}{2}$$

$$= \frac{-4 \pm \sqrt{48}i}{2}$$

$$= \frac{-4 \pm 4\sqrt{3}i}{2}$$

$$= -2 \pm \sqrt{3}i$$

$$= -2 \pm \sqrt{3}i$$

$$\text{(imaginary roots)}$$

$$\text{(not possible)}$$

(imaginary roots)
(not possible)

$$\boxed{x = 4}$$

$$y = \frac{32}{x^2} = \frac{32}{16}$$

$$\boxed{y = 2}$$

$$\boxed{x = 4}$$

$$\boxed{y = 2}$$

$$\int \frac{2x^2 - x - 7}{(x+2)^2 (x^2 + 2x + 5)} dx$$

First we consider:

$$\frac{2x^2 - x - 7}{(x+2)^2 (x^2 + 2x + 5)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{Cx + D}{x^2 + 2x + 5}$$

$$2x^2 - x - 7 = A(x+2)(x^2 + 2x + 5) + B(x^2 + 2x + 5) + (Cx + D)(x+2)^2$$

$$\text{Let } x+2=0$$

$$x = -2$$

$$2(-2)^2 - (-2) - 7 = A(0) + (Cx + D)(0) + B(4 - 4 + 5)$$

$$39 = B(5)$$

$$B = \frac{39}{5}$$

$$2x^2 - x - 7 = A(x^3 + 2x^2 + 5x + 2x^2 + 4x + 10) + B(x^2 + 2x + 5) + (Cx + D)(x^2 + 4x + 4)$$

$$2x^2 - x - 7 = A(x^3 + 4x^2 + 9x + 10) + B(x^2 + 2x + 5) + Cx^3 + 4Cx^2 + 4Cx + Dx^2 + 4Dx + 4D$$

Comparing coefficients:

$$A + C = 0$$

$$A = -C$$

$$4A + B + 4C + D = 0$$

$$4A + B + 4(-A) + D = 0$$

$$B + D = 0$$

$$D = -\frac{B}{5}$$

$$10A + 5B + 4D = -7$$

$$10A + 5\left(\frac{39}{5}\right) + 4\left(\frac{-7}{5}\right) = -7$$

$$10A = -7 - \frac{4 \times 7}{5} + \frac{5 \times 39}{5}$$

$$10A = \frac{-78}{5} = -\frac{39}{25}$$

$$C = \frac{39}{25}$$

$$A = -\frac{39}{25}$$

Q. No. 4 (Page 2)

$$\frac{2x^2 - x - 7}{(x+2)^2(x^2+2x+5)} = \frac{-39}{25(x+2)} + \frac{3}{5(x+2)^2} + \frac{\frac{39}{25}x + \frac{7}{5}}{x^2+2x+5}$$

$$= \frac{-39}{25(x+2)} + \frac{3}{5(x+2)^2} + \frac{39x+35}{25(x^2+2x+5)}$$

Taking integral on both sides.

$$= -\frac{39}{25} \int \frac{dx}{(x+2)} + \frac{3}{5} \int \frac{dx}{(x+2)^2} + \frac{39}{25} \int \frac{x + \frac{35}{39}}{x^2+2x+5} dx$$

$$= -\frac{39}{25} \ln(x+2) + \frac{3}{5} \frac{(x+2)^{-2+1}}{-2+1} + \frac{39}{25} \int \frac{2x + \frac{70}{39}}{x^2+2x+5} dx$$

$$= -\frac{39}{25} \ln(x+2) - \frac{3}{5(x+2)} + \frac{39}{25} \int \frac{2x+2}{x^2+2x+5} + \frac{39}{25} \int \frac{\frac{70}{39}-2}{(x^2+2x+1)+4}$$

$$= -\frac{39}{25} \ln(x+2) - \frac{3}{5(x+2)} + \frac{39}{25} \ln(x^2+2x+5) + \frac{-4x}{25 \cdot 2} \tan^{-1}\left(\frac{x+1}{2}\right) + c$$

$$= -\frac{39}{25} \ln(x+2) - \frac{3}{5(x+2)} + \frac{39}{25} \ln(x^2+2x+5) - \frac{2}{25} \tan^{-1}\left(\frac{x+1}{2}\right) + c$$

Q. No. 4 (Page 3)

Equation of tangent = ?

$$\frac{x^2}{128} + \frac{y^2}{18} = 1$$

$$3x + 8y + 1 = 0$$

$$\text{slope} = -\frac{a}{b} = -\frac{3}{8}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a^2 = 128 \quad b^2 = 18$$

Let eq of tangent be:

$$y = mx + c$$

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

$$y = -\frac{3}{8}x \pm \sqrt{\frac{(128)(9)}{64} + 18}$$

$$y = -\frac{3x}{8} \pm \sqrt{36}$$

$$y = -\frac{3x}{8} \pm 6$$

$$\frac{8y}{8} = -\frac{3x}{8} \pm \frac{48}{8}$$

$$8y = -3x \pm 48$$

$$3x + 8y \pm 48 = 0$$

$$3x + 8y + 48 = 0$$

$$3x + 8y - 48 = 0.$$

Point of contact.

$$3x + 8y + 48 = 0.$$

$$x = \frac{-48 - 8y}{3}$$

$$\frac{\left(\frac{-48 - 8y}{3}\right)^2}{128} + \frac{y^2}{18} = 1$$

Q. No. 4 (Page 4)

$$\frac{8(y+6)^2}{9(128)} + \frac{y^2}{18} = 1$$

$$\frac{1}{16 \times 9} (y^2 + 12y + 36) + \frac{y^2}{18} = 1$$

$$\frac{18(y^2 + 12y + 36) + 16y^2}{16 \times 18 \times 9} = 1$$

$$18y^2 + 216y + 648 + 16y^2 = 288$$
$$34y^2 + 216y + 360 = 0$$

$$18y^2 + 216y + 648 + 144y^2 = 2529$$

$$162y^2 + 216y - 1881 = 0$$

$$y = (2.8) \quad y = -4.1387$$

$$\frac{1}{2 \times 8 \times 9} (y^2 + 12y + 36) + \frac{y^2}{9 \times 2} = 1$$

$$(y^2 + 12y + 36) + 8y^2 = 144$$

$$9y^2 + 12y - 108 = 0$$

$$y = \frac{6 \pm 6\sqrt{22}}{7}$$

$$x = -8 \left(\frac{6-y}{3} \right)$$

$$x = -8 \left(\frac{6 - \frac{6 \pm 6\sqrt{22}}{7}}{3} \right)$$

$$= -8 \left(\frac{42 \pm 6 \pm 6\sqrt{22}}{7} \right)$$

$$= 8 \left(\frac{36 \pm 6\sqrt{22}}{7} \right)$$

$$P(x, y) = P \left(\frac{8(36 \pm 6\sqrt{22})}{7}, \frac{6 \pm 6\sqrt{22}}{7} \right)$$

i- Equation of sides :

 \overline{AB} :

$$\text{slope of } \overline{AB} = m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{6 + 4}{4 + 3} = \frac{10}{7}$$

From point slope form.

$$y - y_1 = m(x - x_1)$$

$$y + 4 = \left(\frac{10}{7}\right)(x + 3)$$

$$7y + 28 = 10x + 30$$

$$10x - 7y - 2 = 0$$

 \overline{AC} :

$$\text{slope of line } \overline{AC} = m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_2 = \frac{-3 + 4}{4 + 3} = \frac{1}{7}$$

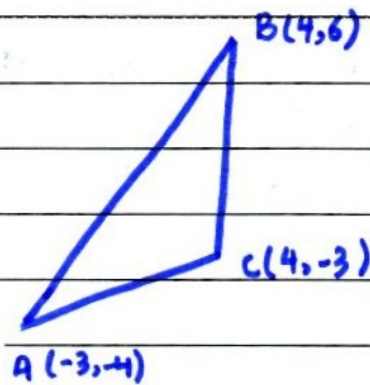
$$y - y_1 = m(x - x_1)$$

$$y + 4 = m(x + 3)$$

$$y + 4 = \left(\frac{1}{7}\right)(x + 3)$$

$$7y + 28 = x + 3$$

$$x - 7y - 25 = 0$$



Q. No. 5 (Page 2)

For interior angle A:

$$\text{Let } \overline{AC} = l_2$$

$$\text{Let } \overline{AB} = l_1$$

$$\theta: l_2 \rightarrow l_1$$

$$m_1 = \frac{10}{7}$$

$$m_2 = \frac{1}{7}$$

$$\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{\left(\frac{10}{7} - \frac{1}{7}\right)}{1 + \left(\frac{10}{7}\right)\left(\frac{1}{7}\right)}$$

$$= \frac{\frac{9}{7}}{1 + \frac{10}{49}} = \frac{\frac{9}{7}}{\frac{59}{49}}$$

$$\tan \theta = \frac{63}{59}$$

$$59$$

$$\theta = \tan^{-1} \left(\frac{63}{59} \right)$$

$$\theta = 46.87^\circ$$

$$\boxed{\angle A = 46.87^\circ}$$

Area of ΔABC :

$$\text{Area of } \Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 4 & 6 & 1 \\ 4 & -3 & 1 \\ -3 & -4 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [4(-3+4) - 6(4+3) + 1(-16-9)]$$

$$= \frac{1}{2} [4 - 42 - 25] = \frac{1}{2} [-63]$$

Q. No. 5 (Page 3)

Since area is +ve so;

$$\text{Area} = \frac{63}{2} \text{ sq units.}$$

d. Distance from $C(4, -3)$ to \overline{AB}

$$\text{Eq of line } \overline{AB} = 10x - 7y - 2 = 0 \quad C(4; -3) = C(x_1, y_1)$$

$$\text{Distance from point to line} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|10(4) - 7(-3) - 2|}{\sqrt{10^2 + (-7)^2}}$$

$$= \frac{|40 + 21 - 2|}{\sqrt{149}}$$

$$= \frac{7}{\sqrt{149}} \text{ units} \cdot \frac{59}{\sqrt{149}} = 1 \text{ unit}$$

$$9x^2 - y^2 - 12x - 2y + 2 = 0$$

$$9x^2 - 12x - y^2 - 2y + 2 = 0$$

$$9\left(x^2 - \frac{12}{9}x\right) - (y^2 + 2y) + 2 = 0$$

$$9\left(x^2 - \frac{4}{3}x\right) - (y^2 + 2y) + 2 = 0$$

$$9\left(x^2 - \frac{4}{3}x + \frac{4}{9} - \frac{4}{9}\right) - (y^2 + 2y + 1 - 1) + 2 = 0$$

$$9\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) - \frac{4}{9} - (y^2 + 2y + 1) + 1 + 2 = 0$$

$$9\left(x - \frac{2}{3}\right)^2 - (y + 1)^2 + 1 + 2 - 4 = 0$$

$$9\left(x - \frac{2}{3}\right)^2 - (y + 1)^2 = 1$$

$$\frac{9\left(x - \frac{2}{3}\right)^2}{1} - \frac{(y + 1)^2}{1} = 1$$

$$\frac{\left(x - \frac{2}{3}\right)^2}{1/9} - \frac{(y + 1)^2}{1} = 1$$

Centre:

$$\text{Let } X = x - \frac{2}{3} \quad Y = y + 1$$

So eq becomes

$$\frac{X^2}{1/9} - \frac{Y^2}{1} = 1 \quad (\text{hyperbola})$$

$$C(X, Y) = C(0, 0)$$

$$X = 0 \quad Y = 0$$

$$\frac{x - 2}{3} = 0 \quad y + 1 = 0$$

$$x = \frac{2}{3} \quad y = -1$$

$$C\left(\frac{2}{3}, -1\right)$$

Foci:

$$F(X, Y) = F(\pm c, 0)$$

$$a^2 = \frac{1}{9} \quad b^2 = 1$$

$$c^2 = a^2 + b^2$$

$$c^2 = \frac{1}{9} + 1 = \frac{10}{9}$$

$$c = \pm \frac{\sqrt{10}}{3} \quad c = \frac{\sqrt{10}}{3}$$

$$F(X, Y) = F\left(\pm \frac{\sqrt{10}}{3}, 0\right)$$

$$X = \pm \frac{\sqrt{10}}{3} \quad Y = 0$$

$$x - \frac{2}{3} = \pm \frac{\sqrt{10}}{3} \quad y + 1 = 0$$

$$x = \frac{2 \pm \sqrt{10}}{3} \quad y = -1$$

$$F\left(\frac{2 \pm \sqrt{10}}{3}, -1\right)$$

Vertices:

$$V(X, Y) = V(\pm a, 0)$$

$$a^2 = \frac{1}{9} \quad a = \pm \frac{1}{3}$$

$$V(X, Y) = V\left(\pm \frac{1}{3}, 0\right)$$

Q. No. 6 (Page 3)

$$X = \pm \frac{1}{3}$$

$$Y = 0$$

$$x - \frac{2}{3} = \pm \frac{1}{3}$$

$$x - \frac{2}{3} = -\frac{1}{3}$$

$$y + 1 = 0$$

$$x = 1$$

$$x = \frac{1}{3}$$

$$y = -1$$

$$V(1, -1)$$

$$V\left(\frac{1}{3}, -1\right)$$

Eccentricity:

$$c = ae$$

$$e = \frac{c}{a} = \frac{\sqrt{10}/3}{1/3} = \sqrt{10}$$

$$e = \sqrt{10}$$

Direction:

$$X = \pm \frac{a}{e}$$

$$X = \pm \frac{1/3}{\sqrt{10}}$$

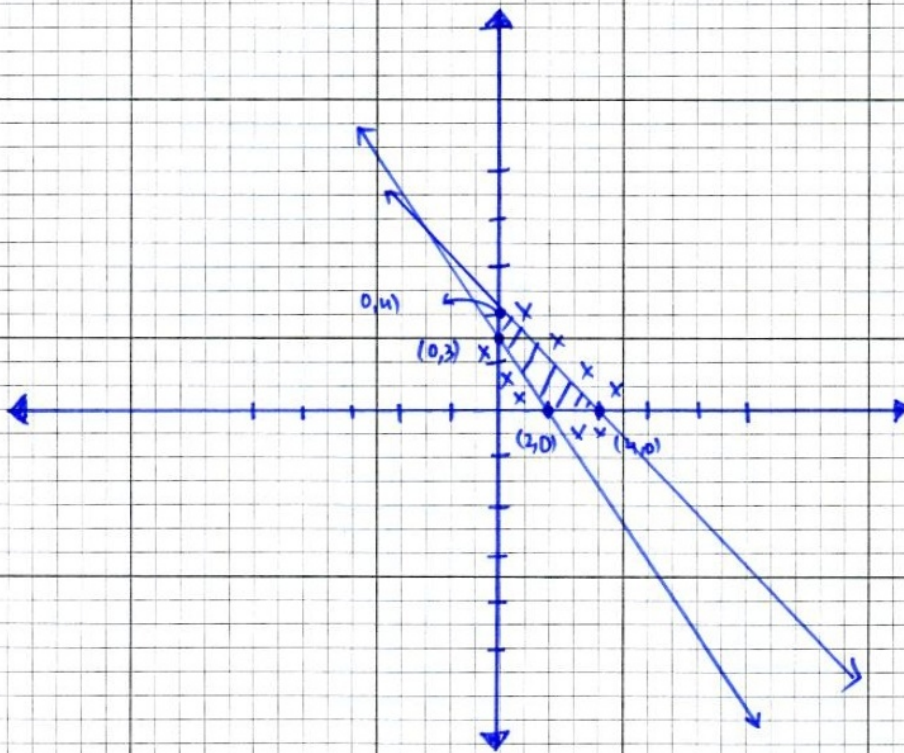
$$X = \pm \frac{1}{3\sqrt{10}}$$

$$x - \frac{2}{3} = \pm \frac{1}{3\sqrt{10}}$$

$$x = \frac{2}{3} \pm \frac{1}{3\sqrt{10}}$$

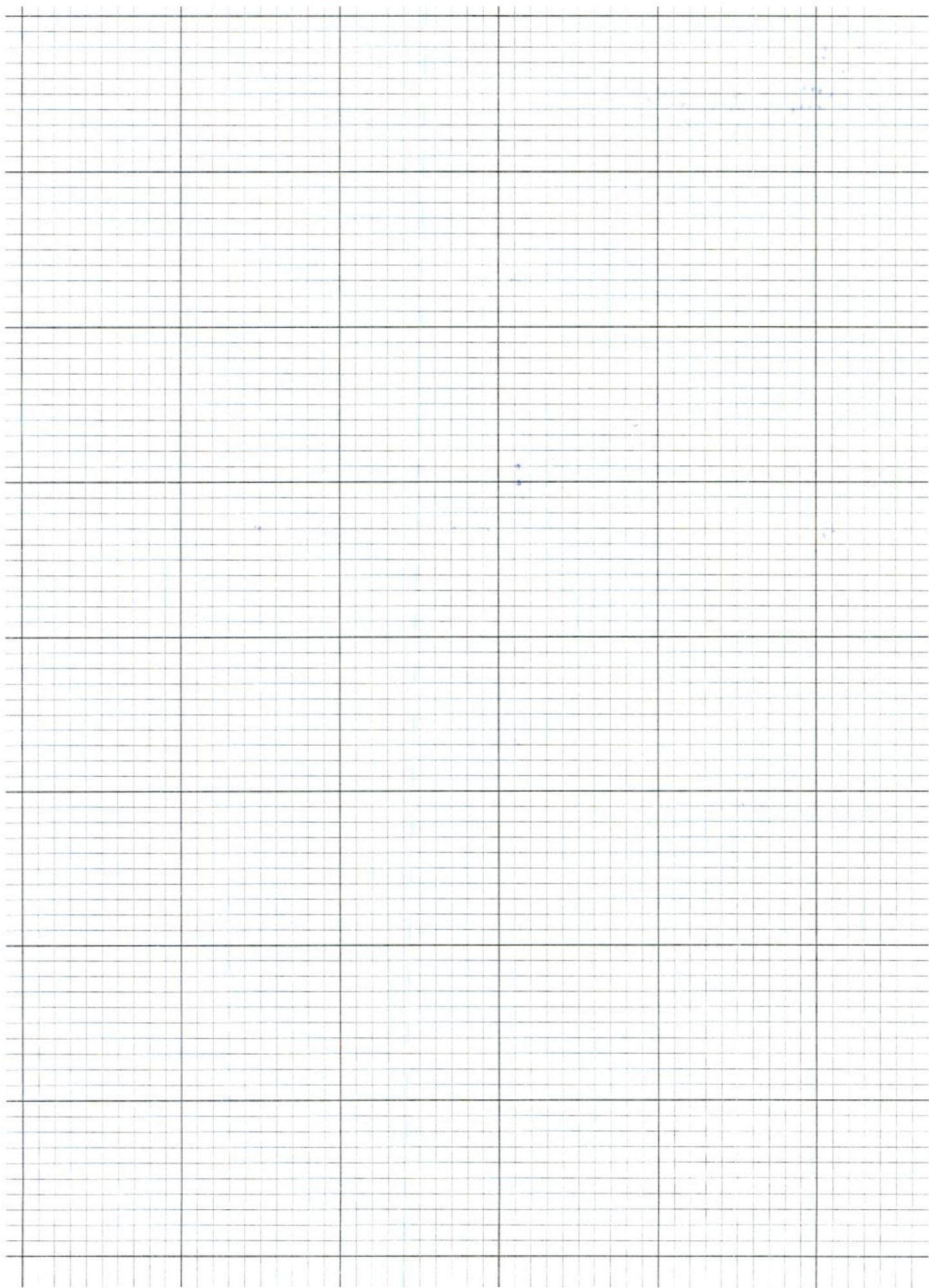
$$x = \frac{2\sqrt{10} \pm 1}{3\sqrt{10}}$$

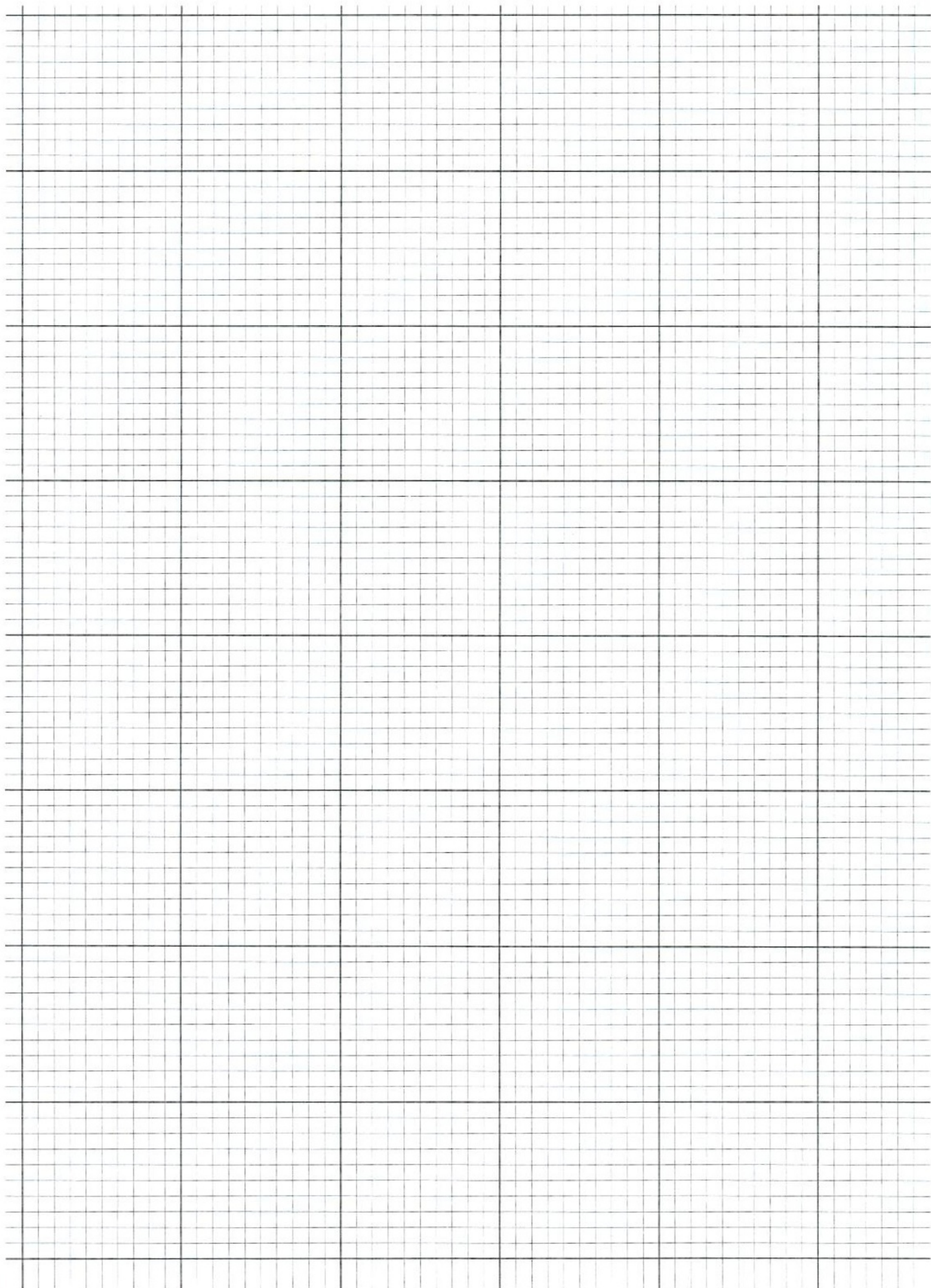
Q:2
(viii) Graph



1 square along $x = 1$ unit .

1 square along $y = 1$ unit .





$$y = (-\sqrt{2})x$$

$$y = (-1 + \sqrt{2})x$$

$$y = (-1 - \sqrt{2})x$$

$$f'(x) = \frac{x}{x^2+1}$$

$$x^2 y + y = x$$

$$x^2 y - x = -y$$

x

$$y = \ln(x^2+1)$$

$$y = \frac{1}{2} \ln \frac{2x}{x^2+1}$$

$$\frac{1}{2} \ln(x^2+1)$$

$$\begin{vmatrix} 3 & -1 & -2 \\ 5 & d & -3 \\ 2 & 1 & -2 \end{vmatrix}$$

$$3(1-2d+3) + 1(-10+6) - 2(5-2d)$$

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$$\sqrt{\sin^2 d + \cos^2 d + t^2} = 3$$

$$\sqrt{t^2} = 3 \quad \sqrt{1+t^2} = 3$$

$$t = 3 \quad 1+t^2 = 9$$

$$t^2 = 9 - 1 = 8$$

$$f(x) = f^{-1}(3x)$$

$$y^2 = (\sqrt{x+5})^2$$

$$y^2 = x+5$$

$$y^2 + 5 = x$$

$$y^2 + 5 = f^{-1}(x)$$

$$9x^2 + 5$$

$$1 + \frac{dy}{dx} = \cos(x+y) \times \left(1 + \frac{dy}{dx}\right)$$

$$1 + \frac{dy}{dx}$$

$$2 \left[\frac{x^2}{2} - x \right]_0^2 = 1$$

$$2 \left[\frac{2^2}{2} - 2k \right] = 1$$

$$[2 - 2k] = \frac{1}{2}$$

$$2 - \frac{1}{2} = 2k$$

$$x^2 + y^2 + 2x - 4y + \frac{1}{3} = 0$$

$$4 + 1 + 4 - 4 + \frac{1}{3} = 0$$

$$\begin{vmatrix} 3 & -1 & -2 \\ 5 & \alpha & -3 \\ 2 & 1 & -2 \end{vmatrix}$$

$$3(-2\alpha + 3) + 1(-10 + 6) - 2(5 - 2\alpha)$$

$$3 - 6\alpha + 9 - 4 - 10 + 4\alpha = 0$$

$$y = \frac{x}{x^2 + 1}$$

$$x^2 y + y = x$$

$$x^2 y - x = y$$

$$2x + 2 = 0$$

$$x = -\frac{2}{2} = -1$$

$$1 - 2 - 3$$

$$f'(x) = e^{\ln \sin x} \times \frac{1}{\sin x} \times \cos x$$

$$= e^{\ln(\sin x)}$$

$$\sin x = y$$

$$dy = \cos x dx$$

$$\int_0^{x/2} e^y y \left| e^{\sin x} \right|_0^{x/2}$$

$$e^1 - e^0$$

$$e - 1$$

$$\begin{vmatrix} 3 & -1 & -2 \\ 5 & \alpha & -3 \\ 2 & 1 & -2 \end{vmatrix} = 0$$

$$3(-2\alpha + 3) + 1(-10 + 6) - 2(5 - 2\alpha) = 0$$

$$3 - 6\alpha + 9 - 10 + 6 - 10 + 4\alpha = 0$$

$$-2\alpha + 5 = 5$$

$$3(6) + 1(4) - 2(3)$$

$$1 + \frac{dy}{dx} = \cos(x+y) \times \left(1 + \frac{dy}{dx}\right)$$

$$3(4+3) + 1(4) - 2(5+1)$$

$$21 - 4 - 21$$

$$\frac{dy}{dx} [1 - \cos(x+y)] = \cos(x+y) - 1$$

$$= \frac{-[1 - \cos(x+y)]}{[1 - \cos(x+y)]}$$

$$v = \frac{x}{x^2 + 1}$$

$$x^2 y + y = x$$

$$x^2 y - x = -y$$

$$y = x - x^2 y$$

$$y = x(1 - xy)$$

$$\left(-\frac{\pi}{\sqrt{2}}\right) \left(-\frac{k}{3}\right) = -1$$

$$\frac{\pi}{3\sqrt{2}} k = -1$$

$$\frac{x}{2} + \frac{y}{-4} = 1$$

$$\frac{x}{2} - \frac{y}{4} = 1$$

$$2x - y = 4$$

$$2x - y - 4 = 0$$

$$2x + y = 0$$

$$x = -1$$