

Q. No. 2 Part (i) (Page 1)

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin^2 \theta}$$

$$= \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{1 - \cos^2 \theta} \quad \begin{array}{l} \because \sin^2 \theta + \cos^2 \theta = 1 \\ \sin^2 \theta = 1 - \cos^2 \theta \end{array}$$

$$= \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{(1 + \cos \theta)(1 - \cos \theta)}$$

$$= \lim_{\theta \rightarrow 0} \frac{1}{1 + \cos \theta}$$

$$= \frac{1}{1 + \cos 0}$$

$$= \frac{1}{1 + 1}$$

$$= \frac{1}{2}$$

$$\boxed{\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin^2 \theta} = \frac{1}{2}}$$

Q. No. 2 Part (ii) (Page 1)

CONTINUOUS AT $x=1$

$f(x)$	$\lim_{x \rightarrow 1} f(x)$
$f(x) = 2(k-1)x$	$f(x) = \frac{3x^2-3}{x-1}$
$f(1) = 2(k-1)1$	$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{3x^2-3}{x-1}$
$= 2(k-1)$	$= \lim_{x \rightarrow 1} \frac{3(x^2-1)}{x-1}$
	$= \lim_{x \rightarrow 1} \frac{3(x+1)(x-1)}{x-1}$
	$= \lim_{x \rightarrow 1} 3(x+1)$
	$= 3 \lim_{x \rightarrow 1} (x+1)$
	$= 3 [1+1]$
	$= 3(2)$
	$= 6$

if $f(x)$ is continuous at $x=1$

$$f(1) = \lim_{x \rightarrow 1} f(x)$$

$$2(k-1) = 6$$

$$k-1 = 3$$

$$k = 3+1$$

$$k = 4$$

Q. No. 2 Part (ii) (Page 2)

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Q. No. 2 Part (iii) (Page 1)

$$f(x) = x^3 - 5x + 1$$

differentiate w.r.t x

$$\begin{aligned} f'(x) &= \frac{d}{dx}(x^3 - 5x + 1) \\ &= 3x^2 - 5(1) + 0 \\ &= 3x^2 - 5 \end{aligned}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

1st Iteration

$$x_0 = 0.5$$

$$\begin{aligned} f(x_0) &= f(0.5) \\ &= (0.5)^3 - 5(0.5) + 1 \\ &= \frac{1}{8} - \frac{5}{2} + 1 \\ &= -\frac{11}{8} \end{aligned}$$

$$\begin{aligned} f'(x_0) &= 3(0.5)^2 - 5 \\ &= 3\left(\frac{1}{4}\right) - 5 \\ &= -\frac{17}{4} \end{aligned}$$

$$\begin{aligned} x_{0+1} &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ &= 0.5 - \frac{\left(-\frac{11}{8}\right)}{\left(-\frac{17}{4}\right)} \\ &= \frac{3}{17} \\ x_1 &= 0.1765 \end{aligned}$$

2nd Iteration

$$x_1 = 0.1765$$

$$\begin{aligned} f(x_1) &= f(0.1765) \\ &= (0.1765)^3 - 5(0.1765) + 1 \\ &= 0.122998 \end{aligned}$$

$$\begin{aligned} f'(x_1) &= f'(0.1765) \\ &= 3(0.1765)^2 - 5 \\ &= -4.906543 \end{aligned}$$

$$\begin{aligned} x_{1+1} &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ x_2 &= 0.1765 - \frac{0.122998}{-4.906543} \\ &= 0.2015 \end{aligned}$$

$$x_1 = 0.1765, x_2 = 0.2015$$

~~PART (a)~~

$$\vec{r}(t) = t^3 \underline{i} + e^{-2t} \underline{j} + \sin 2t \underline{k}$$

differentiate w.r.t t

$$\frac{d\vec{r}}{dt} = \frac{d}{dt} t^3 \underline{i} + \frac{d}{dt} e^{-2t} \underline{j} + \frac{d}{dt} \sin 2t \underline{k}$$

$$\begin{aligned} \vec{v}(t) &= 3t^2 \underline{i} + e^{-2t} \frac{d}{dt} (-2t) \underline{j} + \cos 2t \frac{d}{dt} (2t) \underline{k} \\ &= 3t^2 \underline{i} + e^{-2t} [-2] \underline{j} + \cos 2t [2] \underline{k} \\ &= 3t^2 \underline{i} - 2e^{-2t} \underline{j} + 2 \cos 2t \underline{k} \end{aligned}$$

$$\vec{v}(t) = 3t^2 \underline{i} - 2e^{-2t} \underline{j} + 2 \cos 2t \underline{k}$$

~~PART (b)~~

Acceleration vector

$$\vec{a}(t) = \frac{d\vec{v}}{dt}$$

$$= \frac{d}{dt} [3t^2 \underline{i} - 2e^{-2t} \underline{j} + 2 \cos 2t \underline{k}]$$

$$= 3 \frac{d}{dt} [t^2] \underline{i} - 2 \frac{d}{dt} [e^{-2t}] \underline{j} + 2 \frac{d}{dt} [\cos 2t] \underline{k}$$

$$= 3(2t) \underline{i} - 2e^{-2t} (-2) \underline{j} + 2(-\sin 2t) \frac{d}{dt} (2t) \underline{k}$$

$$= 6t \underline{i} + 4e^{-2t} \underline{j} - 2 \sin 2t (2) \underline{k}$$

$$\vec{a}(t) = 6t \underline{i} + 4e^{-2t} \underline{j} - 4 \sin 2t \underline{k}$$

Q. No. 2 Part (iv) (Page 2)

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First Principle Method

$$y = (2x+3)^{3/2}$$

$$y + \delta y = [2(x + \delta x) + 3]^{3/2}$$

$$= [2x + 2\delta x + 3]^{3/2}$$

$$\delta y = [2x + 2\delta x + 3]^{3/2} - y$$

$$= [2x + 2\delta x + 3]^{3/2} - (2x + 3)^{3/2}$$

$$= [2x + 3 + 2\delta x]^{3/2} - (2x + 3)^{3/2}$$

$$= (2x + 3)^{3/2} \left[1 + \frac{2\delta x}{2x + 3} \right]^{3/2} - (2x + 3)^{3/2}$$

$$= (2x + 3)^{3/2} \left[\left(1 + \frac{2\delta x}{2x + 3} \right)^{3/2} - 1 \right]$$

$$= (2x + 3)^{3/2} \left[1 + \frac{3}{2} \left(\frac{2\delta x}{2x + 3} \right) + \frac{3}{2} \frac{(3/2 - 1)}{2!} \left(\frac{2\delta x}{2x + 3} \right)^2 + \frac{3/2(3/2 - 1)(3/2 - 2)}{3!} \left(\frac{2\delta x}{2x + 3} \right)^3 + \dots \right]$$

$$= (2x + 3)^{3/2} \left[1 + \frac{3}{2} \left(\frac{2\delta x}{2x + 3} \right) + \frac{3}{8} \frac{(4)(\delta x)^2}{(2x + 3)^2} - \frac{1}{16} \frac{(\delta x)^3}{(2x + 3)^3} + \dots \right]$$

$$= (2x + 3)^{3/2} \left[\frac{3}{2} \left(\frac{2\delta x}{2x + 3} \right) + \frac{3}{2} \frac{(\delta x)^2}{(2x + 3)^2} - \frac{1}{16} \frac{(\delta x)^3}{(2x + 3)^3} + \dots \right]$$

$$= (2x + 3)^{3/2} \delta x \left[\frac{3}{2} \left(\frac{2}{2x + 3} \right) + \frac{3}{2} \frac{\delta x}{(2x + 3)^2} - \frac{1}{16} \frac{(\delta x)^2}{(2x + 3)^3} + \dots \right]$$

$$\frac{\delta y}{\delta x} = \frac{(2x + 3)^{3/2} \delta x}{\delta x} \left[\frac{3}{2} \left(\frac{2}{2x + 3} \right) + \frac{3}{2} \frac{\delta x}{(2x + 3)^2} - \frac{1}{16} \frac{(\delta x)^2}{(2x + 3)^3} + \dots \right]$$

$$= (2x + 3)^{3/2} \left[\frac{3}{2} \left(\frac{2}{2x + 3} \right) + \frac{3}{2} \frac{\delta x}{(2x + 3)^2} - \frac{1}{16} \frac{(\delta x)^2}{(2x + 3)^3} + \dots \right]$$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = (2x + 3)^{3/2} \left[\frac{3}{2} \left(\frac{2}{2x + 3} \right) + \frac{3}{2} \frac{(0)}{(2x + 3)^2} - \frac{1}{16} \frac{(0)^2}{(2x + 3)^3} + \dots \right]$$

$$\frac{dy}{dx} = (2x + 3)^{3/2} \left[\frac{3}{2} \left(\frac{2}{2x + 3} \right) + 0 \right]$$

Q. No. 2 Part (ix) (Page 2)

$$= (2x+3)^{3/2} \left(\frac{3}{2}\right) \left(\frac{2}{2x+3}\right)$$

$$= 3(2x+3)^{3/2-1}$$

$$= 3(2x+3)^{1/2}$$

$$\frac{dy}{dx} = 3(2x+3)^{1/2}$$

Q. No. 2 Part (v) (Page 1)

$$\int (3t+1)(t-1)dt$$

$$\begin{array}{l} (3t+1)(t-1) \\ 3t^2 - 3t + t - 1 \end{array}$$

$$= \int (3t^2 - 3t + t - 1)dt$$

$$= \int (3t^2 - 2t - 1)dt$$

$$= 3 \int t^2 dt - 2 \int t dt - \int dt$$

$$= \frac{3t^3}{3} - \frac{2t^2}{2} - t + c$$

$$= t^3 - t^2 - t + c$$

Q. No. 2 Part (vi) (Page 1)

$$f(x) = 3x^2 - 2x$$

Area under the graph

$$= \int_1^3 f(x) dx$$

$$= \int_1^3 (3x^2 - 2x) dx$$

$$= 3 \int_1^3 x^2 dx - 2 \int_1^3 x dx$$

$$= \left. \frac{3x^3}{3} - \frac{2x^2}{2} \right|_1^3$$

$$= \left. x^3 - x^2 \right|_1^3$$

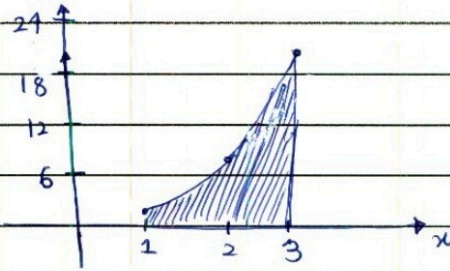
$$= [3^3 - 1^3] - [3^2 - 1^2]$$

$$= [27 - 1] - [9 - 1]$$

$$= 26 - 8$$

$$= 18$$

Area under the graph = 18



Q. No. 2 Part (vii) (Page 1)

$$3x^2 - 2xy - 8y^2$$

~~(PART a)~~

$$3x^2 - 2xy - 8y^2 = 0$$

$$3x^2 - 6xy + 4xy - 8y^2 = 0$$

$$3x(x - 2y) + 4y(x - 2y) = 0$$

$$(3x + 4y)(x - 2y) = 0$$

Pair of straight lines:

$$3x + 4y = 0$$

$$x - 2y = 0$$

~~(PART b)~~

$$\left. \begin{array}{l} 3x^2 - 2xy - 8y^2 = 0 \\ ax^2 + 2hxy + by^2 = 0 \end{array} \right\} \text{compare}$$

$$a = 3 \quad 2h = -2 \quad b = -8$$

$$h = -1$$

$$\tan \theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

$$= \frac{2\sqrt{(-1)^2 - 3(-8)}}{3 - 8}$$

$$= \frac{10}{-5}$$

$$= -2$$

$$\theta = \tan^{-1}(-2)$$

$$\theta = -63.4349 + 180$$

$$= 116.565^\circ$$

Q. No. 2 Part (viii) (Page 1)

$$A(5,4), B(2,1), C(7,3)$$

Area of triangle

$$= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 5 & 4 & 1 \\ 2 & 1 & 1 \\ 7 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \{ 5(1-3) - 4(2-7) + 1(6-7) \}$$

$$= \frac{1}{2} \{ 5(-2) - 4(-5) + 1(-1) \}$$

$$= \frac{1}{2} \{ -10 + 20 - 1 \}$$

$$= \frac{1}{2} (9)$$

$$= \frac{9}{2}$$

$$= 4.5$$

Q. No. 2 Part (x) (Page 1)

$$\int \frac{e^x}{e^{2x} + 1} dx$$

$$\text{Let } u = e^x$$
$$du = e^x dx$$

$$\int \frac{e^x}{e^{2x} + 1} dx$$

$$= \int \frac{1}{u^2 + 1} du$$

$$= \tan^{-1} u + c$$

$$\text{put } u = e^x$$

$$= \tan^{-1} e^x + c$$

$$\int \frac{e^x}{e^{2x} + 1} dx = \tan^{-1} e^x + c$$

PARTIAL DIFFERENTIATION

~~(PART a)~~

$$f(x, y) = \sqrt{x^3 + y^3}$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \sqrt{x^3 + y^3}$$

$$= \frac{\partial}{\partial x} (x^3 + y^3)^{1/2}$$

$$= \frac{1}{2} (x^3 + y^3)^{1/2 - 1} \frac{\partial}{\partial x} (x^3 + y^3)$$

$$= \frac{1}{2} (x^3 + y^3)^{-1/2} [3x^2 + 0]$$

$$= \frac{3x^2 (x^3 + y^3)^{-1/2}}{2}$$

$$= \frac{3x^2 (x^3 + y^3)^{-1/2}}{2}$$

$$\frac{\partial f}{\partial x} = \frac{3x^2}{2\sqrt{x^3 + y^3}}$$

~~(PART b)~~

$$f(x, y) = \sqrt{x^3 + y^3}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} (x^3 + y^3)^{1/2}$$

$$= \frac{1}{2} (x^3 + y^3)^{1/2 - 1} \frac{\partial}{\partial y} [x^3 + y^3]$$

$$= \frac{1}{2} (x^3 + y^3)^{-1/2} \left[\frac{\partial x^3}{\partial y} + \frac{\partial y^3}{\partial y} \right]$$

$$= \frac{1}{2} (x^3 + y^3)^{-1/2} [0 + 3y^2]$$

Q. No. 2 Part (xi) (Page 2)

$$= \frac{3y^2}{2} (x^3+y^3)^{-1/2}$$

$$\frac{\partial f}{\partial y} = \frac{3y^2}{2(x^3+y^3)^{1/2}}$$

Q. No. 2 Part (xii) (Page 1)

differentiation

$$x^3 + 3x^2y^2 + y^3 = 0$$

differentiate w.r.t x

$$\frac{d}{dx} x^3 + 3 \frac{d}{dx} (x^2y^2) + \frac{d}{dx} (y^3) = 0$$

$$3x^2 + 3 \left[x^2 \frac{dy^2}{dx} + y^2 \frac{dx^2}{dx} \right] + 3y^2 \frac{dy}{dx} = 0$$

$$3x^2 + 3 \left[x^2 2y \frac{dy}{dx} + y^2 (2x) \right] + 3y^2 \frac{dy}{dx} = 0$$

$$3x^2 + 6x^2y \frac{dy}{dx} + 6y^2x + 3y^2 \frac{dy}{dx} = 0$$

$$6x^2y \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = -3x^2 - 6xy^2$$

$$(6x^2y + 3y^2) \frac{dy}{dx} = -3x^2 - 6xy^2$$

$$\frac{dy}{dx} = \frac{-3x^2 - 6xy^2}{6x^2y + 3y^2}$$

$$= \frac{3[-x^2 - 2xy^2]}{3[2x^2y + y^2]}$$

$$\frac{dy}{dx} = \frac{-x^2 - 2xy^2}{2x^2y + y^2}$$

Q. No. 3 (Page 1)

$$f(x) = \frac{1}{8}x^3 + \frac{1}{4}x^2 + \frac{2}{x^2} + \frac{1}{2}$$

$$y = \frac{1}{8}x^3 + \frac{1}{4}x^2 + \frac{2}{x^2} + \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{1}{8}(3x^2) + \frac{1}{4}(2x) + 2(-2x^{-3}) + 0$$

$$= \frac{3}{8}x^2 + \frac{1}{2}x - \frac{4}{x^3}$$

Ordinate when $x=2$

$$y = \frac{1}{8}x^3 + \frac{1}{4}x^2 + \frac{2}{x^2} + \frac{1}{2}$$

$$= \frac{1}{8}(2)^3 + \frac{1}{4}(2)^2 + \frac{2}{(2)^2} + \frac{1}{2}$$

$$= \frac{1}{8}(8) + \frac{1}{4}(4) + \frac{2}{4} + \frac{1}{2}$$

$$= 3$$

Slope m

$$(x_1, y_1) = (2, 3)$$

$$m = \frac{dy}{dx} \Big|_{(2,3)}$$

$$= \frac{3}{8}x^2 + \frac{1}{2}x - \frac{4}{x^3}$$

$$= \frac{3}{8}(2)^2 + \frac{1}{2}(2) - \frac{4}{(2)^3}$$

$$= 2$$

~~Q (PART 2)~~

Equation of Tangent

$$y - y_1 = m(x - x_1)$$

$$m = 2 \quad (x_1, y_1) = (2, 3)$$

$$y - 3 = 2(x - 2)$$

$$y - 3 = 2x - 4$$

$$2x - y - 4 + 3 = 0$$

$$2x - y - 1 = 0$$

$$\boxed{2x - y - 1 = 0}$$

Q. No. 3 (Page 2)

PART B

Equation of normal

$$m = 2 \quad (x_1, y_1) = (2, 3)$$

$$y - y_1 = \frac{-1}{m} (x - x_1)$$

$$y - 3 = \frac{-1}{2} (x - 2)$$

$$2(y - 3) = -(x - 2)$$

$$2y - 6 = -x + 2$$

$$x + 2y - 6 - 2 = 0$$

$$x + 2y - 8 = 0$$

$$\boxed{x + 2y - 8 = 0}$$

PART C

CONTINUITY TEST

$$x = 0$$

(i) $f(x)$ is defined or not

For a function to be continuous at $x = 0$, it must be defined at $x = 0$

$$f(x) = \frac{1}{8}x^3 + \frac{1}{4}x^2 + \frac{2}{x^2} + \frac{1}{9}$$

$$f(0) = \frac{1}{8}(0)^3 + \frac{1}{4}(0)^2 + \frac{2}{(0)^2} + \frac{1}{9}$$

$$= \frac{2}{0} + \frac{1}{9}$$

undefined

The function is not defined at $x = 0$ so it is

discontinuous at $x = 0$

Q. No. 4 (Page 1)

~~(PART a)~~
Median \overline{CM}

Midpoint of AB

$$\begin{aligned}M(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{1 + 7}{2}, \frac{2 + 2}{2} \right) \\&= (4, 2)\end{aligned}$$

Equation of Median \overline{CM}

$$M(4, 2) \quad C(3, 6)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2}{6 - 2} = \frac{x - 4}{3 - 4}$$

$$\frac{y - 2}{4} = \frac{x - 4}{-1}$$

$$-1(y - 2) = 4(x - 4)$$

$$-y + 2 = 4x - 16$$

$$4x + y - 16 - 2 = 0$$

$$4x + y - 18 = 0$$

$$\boxed{4x + y - 18 = 0}$$

Q. No. 4 (Page 2)

(PART B)
ALTITUDE \overline{CD}

Slope of \overline{AB}

A(1,2) B(7,2)

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - 2}{7 - 1}$$

$$= 0$$

Slope of \overline{CD}

Since altitude \overline{CD} is perpendicular to \overline{AB} .

$$m_2 = -\frac{1}{m_1}$$

$$= -\frac{1}{0}$$

$$= \infty$$

Equation of altitude \overline{CD}

\overline{CD} passes through C(3,6) and is perpendicular to horizontal.

$$y - y_1 = m(x - x_1)$$

$$y - 6 = -\frac{1}{0}(x - 3)$$

$$0(y - 6) = -x + 3$$

$$0 = -x + 3$$

$$-x + 3 = 0$$

$$x - 3 = 0$$

$$x - 3 = 0$$

(PART C)
RIGHT BISECTOR of \overline{AB}

Slope of \overline{AB}

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - 2}{7 - 1}$$

$$= 0$$

Slope of bisector

Since right bisector is perpendicular to \overline{AB}

$$m_2 = \frac{-1}{m_1}$$

$$= \frac{-1}{0}$$

Midpoint of \overline{AB}

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{1+7}{2}, \frac{2+2}{2} \right)$$

$$= (4, 2)$$

RIGHT BISECTOR OF \overline{AB}

right bisector passes through $M(4, 2)$ and is vertical

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-1}{0}(x - 4)$$

$$(y - 2) \cdot 0 = x - 4$$

$$x - 4 = 0$$

$$\boxed{x - 4 = 0}$$

Q. No. 5 (Page 1)

$$2xy \, dy - (x^2 + 3y^2) \, dx = 0$$

$$2xy \, dy = (x^2 + 3y^2) \, dx$$

$$\frac{dy}{dx} = \frac{(x^2 + 3y^2)}{2xy}$$

$$= \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$= \frac{x}{2y} + \frac{3y}{2x}$$

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{x}{y} \right) + \frac{3}{2} \left(\frac{y}{x} \right) \quad \text{--- (i)}$$

$$\text{let } u = \frac{y}{x}$$

$$y = ux$$

differentiate with respect to x

$$\frac{dy}{dx} = u + x \frac{du}{dx}$$

Substitute $u = \frac{y}{x}$ and $\frac{dy}{dx} = u + x \frac{du}{dx}$ in eq (i)

$$u + x \frac{du}{dx} = \frac{1}{2u} + \frac{3u}{2}$$

$$\begin{aligned} x \frac{du}{dx} &= \frac{1}{2u} + \frac{3u}{2} - u \\ &= \frac{1 + 3u^2 - 2u^2}{2u} \end{aligned}$$

$$x \frac{du}{dx} = \frac{1 + u^2}{2u}$$

$$\frac{2u}{1 + u^2} \, du = \frac{1}{x} \, dx$$

Integrate both sides.

Q. No. 5 (Page 2)

$$\int \frac{2u}{1+u^2} du = \int \frac{1}{x} dx$$

$$\ln |1+u^2| = \ln x + \ln c$$

put $u = \frac{y}{x}$

$$\ln |1 + \left(\frac{y}{x}\right)^2| = \ln x + \ln c$$

$$\ln \left| 1 + \frac{y^2}{x^2} \right| = \ln x + \ln c$$

$$\ln \left| 1 + \frac{y^2}{x^2} \right| = \ln xc$$

$$1 + \frac{y^2}{x^2} = xc$$

Solving for y

$$\frac{y^2}{x^2} = xc - 1$$

$$y^2 = x^2(xc - 1)$$

$$(y^2)^{1/2} = [x^2(xc - 1)]^{1/2}$$

$$y = \pm x \sqrt{xc - 1}$$

Q. No. 6 (Page 1)

SIMPSON'S RULE

(PART 2)

$$\int_0^2 (3x^2 - 2x) dx$$

$$b=2, a=0, f(x)=3x^2-2x, n=2$$

$$\begin{aligned}\Delta x &= \frac{b-a}{n} \\ &= \frac{2-0}{2} \\ &= \frac{2}{2} \\ &= 1 \\ &= 0.5\end{aligned}$$

$$\Delta x = 0.5$$

x	x_0	x_1	x_2	x_3	x_4
	= 0	= 0.5	= 1	= 1.5	= 2
$f(x)$	0	-0.25	1	3.75	8

$$\begin{aligned}S_4 &= \frac{\Delta x}{3} [f_0 + 4(f_1 + f_3) + 2(f_2) + f_4] \\ &= \frac{0.5}{3} [0 + 4(-0.25 + 3.75) + 2(1) + 8] \\ &= \frac{0.5}{3} [24] \\ &= 4\end{aligned}$$

approximate value by Simpson's rule:
4

Q. No. 6 (Page 2)

~~(PART b)~~

$$\int_0^2 (3x^2 - 2x) dx$$

$$= \int_0^2 3x^2 dx - \int_0^2 2x dx$$

$$= 3 \int_0^2 x^2 dx - 2 \int_0^2 x dx$$

$$= 3 \left[\frac{x^3}{3} \right]_0^2 - 2 \left[\frac{x^2}{2} \right]_0^2$$

$$= x^3 \Big|_0^2 - x^2 \Big|_0^2$$

$$= [2^3 - 0^3] - [2^2 - 0^2]$$

$$= 8 - 4$$

$$= 4$$

~~(PART C)~~

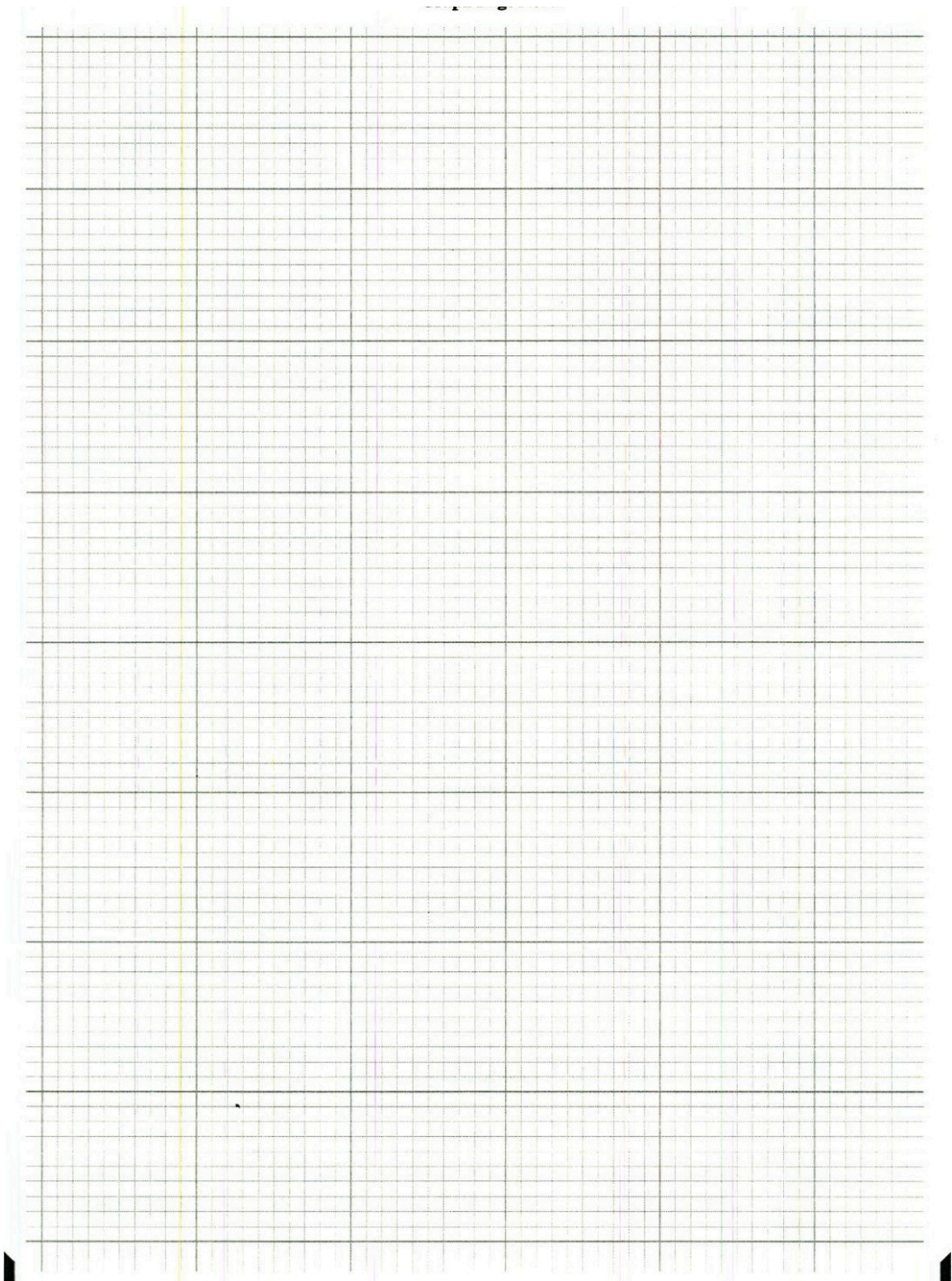
Error = Difference of both values

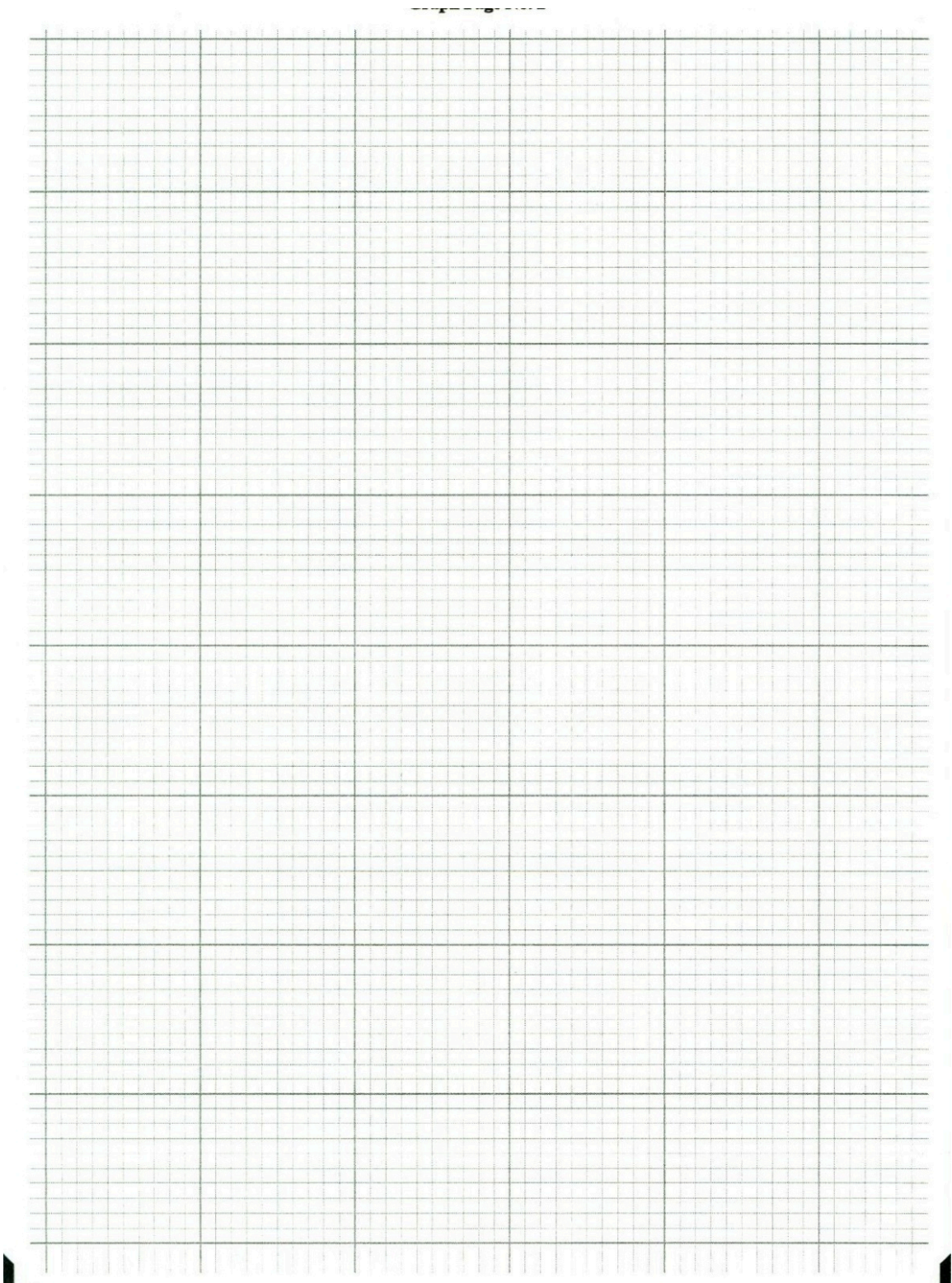
= Actual value - Approximate value

$$= 4 - 4$$

$$= 0$$

no error





$$\frac{2-0}{2n} = \frac{2}{2(2)} = 0.5$$

$$2xydy = (3y^2 + x^2)dx$$

$$\frac{dy}{dx} = \frac{3y^2 + x^2}{2xy}$$

$$= \frac{3y}{2x} + \frac{x}{2y}$$

$$= \frac{3}{2} \left(\frac{y}{x} \right) + \frac{1}{2} \left(\frac{x}{y} \right)$$

$$u = \frac{y}{x}$$

$$y = ux$$

$$\frac{dy}{dx} = u + x \frac{du}{dx}$$

$$u + x \frac{du}{dx} = \frac{3u + 1}{2} \frac{1}{u}$$

$$2 \frac{dy}{dx} = \frac{3u + 1}{2u} - 4$$

$$x \frac{du}{dx} = \frac{3u^2 + 1 - 2u^2}{2u}$$

$$= \frac{1 + u^2}{2u}$$

$$\int \frac{2u}{1+u^2} du = \int \frac{1}{u} dx$$

$$\ln |1+u^2| = \ln x + \ln c$$

$$1+u^2 = xc$$

$$1 + \frac{y^2}{x^2} = xc$$

$$\frac{y^2}{x^2} = xc - 1$$

$$2xy dy - (x^2 + 3y^2) dx = 0$$

$$2xy dy = (x^2 + 3y^2) dx$$

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

$$= \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$= \frac{x}{2y} + \frac{3y}{2x}$$

$$= \frac{1}{2} \left(\frac{x}{y} \right) + \frac{3}{2} \left(\frac{y}{x} \right)$$

$$u = \frac{y}{x}$$

$$y = ux$$

$$\frac{dy}{dx} = u + x \frac{du}{dx}$$

$$u + x \frac{du}{dx} = \frac{1}{2u} + \frac{3u}{2}$$

$$x \frac{du}{dx} = \frac{1}{2u} + \frac{3u}{2} - u$$

$$= \frac{1 + 3u^2 - 2u^2}{2u}$$

$$x \frac{du}{dx} = \frac{1 + u^2}{2u}$$

$$\int \frac{du}{1+u^2} = \int \frac{dx}{x}$$

$$\ln|1+u^2| = \ln|x| + \ln C$$

$$\frac{1 - \cos \theta}{1 + e^{2+\theta} (0+3)}$$

$$\frac{3x^2 - 3}{x - 1}$$

$$\frac{3(x^2 - 1)}{x - 1}$$

$$3(x+1)(x-1)$$

$$3(x+1) = 2(k-1)$$

$$3(1+1) = 2(k-1)$$

$$6 = 2(k-1)$$

$$3 = k-1$$

$$\int dy = \int k dx$$

$$y = \frac{2x^2}{3} + C$$

$$2 \sqrt[3]{(x^3 + y^3)^{2/3}} (3x^2)$$

$$dy = \frac{2}{3} (x^3 + y^3)^{-1/2} (3x^2)$$

$$3x^2 + 3 \left[\frac{3x^2 y dy}{(x^3 + y^3)^{3/2}} + \frac{y^2 (2x)}{(x^3 + y^3)^{3/2}} + \frac{3y^2 dy}{(x^3 + y^3)^{3/2}} \right]$$

$$3x^2 + 6x^2 y \frac{dy}{dx} + 6xy^2 + 3y^2 \frac{dy}{dx} = 0$$

$$(6x^2 y + 3y^2) \frac{dy}{dx} = -\frac{3x^2 + 6}{6x^2 y + 3}$$

$$\frac{dy}{y} = \frac{-x^2 - 2}{2x^2 y + y} dx$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{-x^2 - 2}{2x^2 + 1}$$

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$$r = \frac{\partial t_i}{\partial t_j} = \frac{\partial t_i}{\partial t_j}$$

$$\int \cot x dx = \ln|\sin x|$$

$$2 \int x dx = 2 \left[\frac{x^2}{2} \right]$$

$$\frac{2}{3} [2^3 - 0^3] = 3k - 9$$

$$4 = 3k - 9$$

$$6 = 3k$$

$$x^3 - 5x + 1$$

$$3x^2 - 5$$

$$3t^2 i + e^{-2t} (-2j) + \cos 2t (2)$$

$$\Rightarrow 3(2t) i - 2(e^{-2t}) j + 2(-\sin 2t) (2)$$

$$= 6t i - 2e^{-2t} j - 4 \sin 2t$$

$$= 6t i - 2e^{-2t} j - 4 \sin 2t$$

$$= 6t i - 2e^{-2t} j - 4 \sin 2t$$

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$$\Delta x = \frac{b-a}{n}$$

$$e^{x+y} \left[\frac{b-a}{n} \right]$$

$$= \frac{2-0}{4}$$

$$= 0.5$$

$$\frac{1}{2} \left| \begin{matrix} e^{5j} [4+3] \\ e^{7i} 0 3 \\ 1 \end{matrix} \right|$$

$$= \frac{1}{2} \left(5(12) - 1(6+7) + 1(6-7) \right)$$

$$= \frac{1}{2} (60 - 13 + 1) = \frac{48}{2} = 24$$

$$M(4, 2) = (3, 6)$$

$$\frac{y-2}{6-2} = \frac{x-4}{3-4}$$

$$\frac{y-2}{4} = \frac{x-4}{-1}$$

$$y-2 = -4(x-4)$$

$$y-2 = -4x + 16$$

$$4x + y - 18 = 0$$